Issue Date: June 27, 2024

Mathematics Proficiency-Based Graduation Hierarchy

Introduction

The exemplar Proficiency-Based Graduation Requirement (PBGR) Hierarchies support equity by providing a cohesive and coordinated vision of student-centered learning across Vermont schools. The hierarchies serve as a foundation for the implementation of standards adopted by the Vermont State Board of Education, Local Comprehensive Assessment Systems, flexible pathways, and personalized learning plans. The Agency of Education recognizes the considerable time and effort that educators and other stakeholders across the state have already put forth developing proficiency-based learning systems and this work is intended to serve as a resource for SU/SDs to consider as they reflect on the key concepts and skills that students should develop within each content area. It is important to note that while there is a separate hierarchy for each disciplinary content area, the hierarchies work together to support student proficiency in those attributes described in a Vermont Portrait of a Graduate (PoG).

Process

The first step in the process of developing PBGR Hierarchies involved the creation of content area PoGs. The Vermont Agency of Education (AOE) did this by reading the Vermont PoG through a content lens, comparing the PoG indicators with national content standards, and identifying those indicators that could best be addressed within that content area (e.g., A Vermont Portrait of a Graduate through the lens of Social Studies). The AOE integrated these content area PoGs, State board-approved national standards, and other research to define what it means to be literate within each disciplinary content area. Each literacy statement was pared down to its essential elements, resulting in one PBGR for each content area. Once the PBGR was developed, the AOE identified the Critical Proficiencies (CPs) that would support the teaching and learning related to that PBGR. The AOE developed Priority Performance Indicators (PPIs) for each CP, based on national standards, research, and input from the field. While the PPIs will be formally assessed to develop evidence of student proficiency, all standards have a place in the curriculum and in student learning. Throughout this process, attention was also given to how transferable skills could be interwoven throughout each PBGR Hierarchy. The construction of these hierarchies was an iterative process, taking on many different stages with educator feedback and internal review being essential components of that process.

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Proficiency-Based Graduation Hierarchy Development Process

The Critical Proficiencies in the mathematics hierarchy represent the five conceptual categories laid out in the Common Core State Standards for Mathematics at the high school level: Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. The PPIs supporting each CP are tied to the CCSS and are clearly annotated. The PPIs were chosen based on careful examination and comparison of the following sources:

- High school mathematics standards from the CCSS,
    - Proficiency-based graduation requirements already written by schools and districts around the state,
- The work of the Essential Math for College and Careers program, and
- SAP/Achieve the Core's Widely Applicable Prerequisites.

In addition to this research, input was taken from the field on initial drafts and used in the final selection of the PPIs. It is important to note that while the PPIs list only content standard clusters, the mathematics practice standards outlined in the CCSS are equally important. The practice standards are:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics, Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

A mathematically literate person is proficient in the areas outlined in the content-based PPIs and is also skilled in the habits and ways of working outlined in the practice standards. Many of the practice standards also connect easily to the Transferable Skills Graduation Requirements.

Mathematics PBGR

The mathematically literate individual can reason quantitatively about patterns and shapes to analyze and solve real-world and hypothetical problems. They can use equations, data, tables, and graphs to validate hypotheses and model situations that individuals encounter throughout their lives.

Critical Proficiency	Critical Proficiency	Critical Proficiency	Critical Proficiency	Critical Proficiency
Quantitative Reasoning	**Algebraic Reasoning**	**Functional Reasoning**	**Geometric Reasoning**	**Statistical Reasoning**
Reason quantitatively and use units to solve problems.	Create, interpret, use, and analyze expressions, equations, and inequalities.	Use linear and non-linear functions to interpret and analyze a variety of contexts.	Apply geometric concepts to solve problems in a variety of contexts.	Interpret and apply statistics and probability to analyze data, justify conclusions, and make inferences.

Priority Performance Indicator	Priority Performance Indicators	Priority Performance Indicators	Priority Performance Indicators	Priority Performance Indicators
Reasoning Quantitatively Reason quantitatively and use units to solve problems. (HSN.Q.A)	**Interpreting Expressions** Interpret the structure of expressions. (HSA.SSE.A)	**Function Notation** Understand the concept of a function and use function notation. (HSF.IF.A)	**Geometric Theorems** Prove geometric theorems. (HSG.CO.C)	**Single Count Data** Summarize, represent, and interpret data on a single count or measurement variable. (HSS.ID.A)
	Creating Equations Create equations that describe numbers or relationships. (HSA.CED.A)	**Analyzing Functions** Analyze linear and non-linear functions using different representations. (HSF.IF.C)	**Right Triangle Trigonometry** Define trigonometric ratios and solve problems involving right triangles. (HSG.SRT.C)	**Summarize Categorical Data** Summarize, represent, and interpret data on two categorical and quantitative variables. (HSS.ID.B.)
	Solving Equations Solve equations and inequalities in one variable. (HSA.REI.B)	**Building Functions** Build a function that models a relationship between two quantities. (HSF.BF.A)		
	Graphing Equations Represent and solve equations and inequalities graphically. (HSA.REI.D)			

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